

Annual Report for Period: 09/2003 - 09/2004

Principal Investigator: Cunefare, Kenneth A.

Organization: GA Tech Res Corp - GIT

Title:

Suppression of Friction-Induced Oscillations Through Use of High-Frequency Dither Signals

Submitted on: 12/24/2005

Award ID: 0101122

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N/AProject Participants

Senior Personnel

Name: Cunefare, Kenneth

Worked for more than 160 Hours: Yes

Contribution to Project:

Name: Ferri, Aldo

Worked for more than 160 Hours: Yes

Contribution to Project:

Post-doc

Graduate Student

Name: Michaux, Michael

Worked for more than 160 Hours: Yes

Contribution to Project:

Michael Michaux participated on the theoretical development related to the project. He has been developing the modeling methods and codes necessary to evaluate the impact of dither on squeal. Michael is a Ph.D. candidate, having passed his qualifier exams in the spring of 2002. Michael was supported by the project with a 1/3 time assistantship. Michael completed his PHD in June of 2005.

Name: Badertscher, Jeffery

Worked for more than 160 Hours: Yes

Contribution to Project:

Jeffery Badertscher is working on the experimental program of the project. He has integrated the torque sensor into the brake squeal dynamometer, and has begun evaluation of the impact of dither control on mean braking torque. Jeff was supported through other resources (TA position, and lab development funds).

Jeff passed his Ph.D. qualifying examinations in the spring of 2003. He completed his PhD proposal presentation in June of 2005.

Name: Dzirasa, Mawuli

Worked for more than 160 Hours: Yes

Contribution to Project:

Mawuli Dzirasa performed extensive experimental evaluations of the efficacy of various dither signal types upon control effectiveness. Mawuli was self-supported. Mawuli graduated in Spring 2002.

Name: Do, Nguyen

Worked for more than 160 Hours: Yes

Contribution to Project:

Nguyen Do was advised by Dr. Ferri in SDOF analytical studies.

Name: Krizan, Nick

Worked for more than 160 Hours: Yes

Contribution to Project:

Nick Krizan developed a simple slider with resonant dither actuator, for experimental analyses of a system with greatly reduced

complexity as compared to the full-up brake caliper/rotor system.

Undergraduate Student

Name: Doan, June

Worked for more than 160 Hours: No

Contribution to Project:

June Doan was an undergraduate special topics student who worked with Mawuli Dzirasa in characterizing the performance of the dither control system. She participated for one semester, and received an Undergraduate Research Assistanceship from the Institute. June graduated in spring of 2002. Currently employed by Goodyear.

Name: Manning, Benjamin

Worked for more than 160 Hours: No

Contribution to Project:

Ben Manning was an undergraduate special topics student working with Jeff Badertscher on assessing conditions that induce squeal. Ben was self-supported. Ben graduated in spring of 2003. He planned to pursue graduate study.

Name: Muhlberger, Emily

Worked for more than 160 Hours: No

Contribution to Project:

Emily Muhlberger participated as an unpaid undergraduate research assistant during the spring semester, 2003. She gained hands-on experience with the experiment. She performed parameter testing for squeal characterization.

Name: Paradiso, Marc

Worked for more than 160 Hours: No

Contribution to Project:

Marc Paradiso participated as an unpaid undergraduate research assistant during the spring semester, 2003. He gained hands-on experience with the experiment. He performed parameter testing for squeal characterization.

Name: Borjas, Ricardo

Worked for more than 160 Hours: Yes

Contribution to Project:

Ricardo Borjas participated with the experimental program as a means to complete the requirement of his 'long internship' from the University of Simon Bolivar, Venezuela. His interest in the project developed from Dr. Cunefare's visit to USB in March of 2002. Ricardo performed extensive testing of the dither waveform impact on performance. NOTE: Ricardo was self-supported.

Name: Ortecho, Alberto

Worked for more than 160 Hours: Yes

Contribution to Project:

Alberto Ortecho participated with the experimental program as a means to complete the requirement of his 'long internship' from the University of Simon Bolivar, Venezuela. His interest in the project developed from Dr. Cunefare's visit to USB in March of 2002. Alberto performed extensive testing of the dither waveform impact on performance. Additionally, he developed designs for alternative actuator embodiments. NOTE: Alberto was self-supported.

Name: Flynn, Michael

Worked for more than 160 Hours: No

Contribution to Project:

Michael Flynn performed an undergraduate research experience project investigating alternative means to implement dither actuators into brake systems. He planned to pursue graduate study.

Technician, Programmer

Other Participant

Research Experience for Undergraduates**Organizational Partners****Other Collaborators or Contacts**

Dr. Wayne Whiteman participated with Dr. Ferri in the work documented in the Dr. Wayne Whiteman 'Stability Analysis of A Vibrational System Subject To Negative Viscous Damping And Displacement-Dependent Dry Friction Damping,' Proceedings of the ASME 2003 Design Engineering Technical Conferences, Chicago, Illinois USA, September 2-6, 2003.

Dr. Mark Donley of MTS Corporation completed a number of simulations of the impact of dither on groan, a lower frequency brake noise than squeal. The question of interest was whether or not dither will be effective against groan. The simulations indicated a reduction in groan vibration amplitudes, but not a complete suppression.

TRW provided a multi-piston brake caliper system for evaluation of the concept. We began developing the test hardware to mount the caliper within our brake dynamometer (Michael Flynn participated in this activity).

GM/Delphi contacted us for information concerning the work, and several divisions entertained white papers related to a custom integration within a Delphi system. Delphi ultimately did not pursue the concept.

Various mass-market publications of the work (Popular Mechanics, MIT Tech Review, etc.) resulted in a number of contacts from various organizations (MARTA, New York Transit Authority, etc.) who have a keen interest in reducing brake squeal in their fleets.

A representative of 'Railroad Friction Products Corp.' contacted us for information on potential applications of the concept within the railroad industry. We provided papers and responses to specific queries.

Activities and Findings

Research and Education Activities: (See PDF version submitted by PI at the end of the report)

Findings: (See PDF version submitted by PI at the end of the report)

Analytical Component

Numerical studies of tangential dither have shown the combinations of dither frequency and amplitude for which self-excited stick-slip oscillations are suppressed. It was found that the dither amplitude required to suppress the sustained oscillations decreases with dither frequency; i.e., high frequency dither signals (relative to the system resonant frequency) seem to be the most effective.

It was found that, for a given dither frequency, the dither amplitude required to suppress sustained oscillations follows a linear relationship with the ratio of static coefficient of friction over dynamic coefficient of friction, μ_s/μ_d . In other words, as μ_s/μ_d approaches 1, the required amplitude of the tangential dither signal decreases.

It was found that small changes in system stability could be realized when the system was in a parameter range near a stability boundary. This investigation is ongoing and has yet to be compared qualitatively with the observed squeal suppression characteristics exhibited by the experimental brake dynamometer system.

Experimental Component

The results of torque testing to date indicate that a continuous dither signal of 23 kHz and varying force amplitudes reduces the effective braking torque by at most 3%. These results suggest that perhaps the Hess and co-workers model is a good starting point but that it overestimates the impact that dither control has on braking torque.

The impact of dither control on the effective braking torque is directly related to the force amplitude of the dither signal. This result could prove to be the most significant if a threshold value of dither force amplitude is established for torque reduction.

Training and Development:

To date, 5 graduate students have participated in the project since the funding date. Of these students, one MS student has completed his MS thesis, gaining experience in conducting and reporting upon a significant experimental effort. A second MS student is in progress, and has gained detailed experimental planning and execution experience, as he is principally responsible for conducting the experimental effort and the inevitable spin-off queries that the work engenders. This student passed his PhD qualifying examinations in Spring 2003. Further, this student has gained experience with student supervision, having participated as TA in four undergraduate lab classes that have used the test rig, training and supervising 4 undergraduate special topics students, and, supervising the visiting students from Universidad Simon Bolivar. Finally, a Ph.D. student has gained experience in theoretical research, gaining skills in literature review and comparison, modeling and simulations. Two additional MS students have been engaged in various aspects of analytical and experimental programs.

Outreach Activities:

Presentations were made to Akebono Corp. and General Motors. White papers were provided to Delphi and General Motors for potential extensions of the work.

In the 2002 reporting period, a presentation of the project and experimental results was made at Universidad Simon Bolivar, Caracas, VZ, in March, 2002. The presentation was invited, and delivered at a student-organized colloquium related to the globalization of engineering. This presentation resulted in two USB students studying in my lab from July through December 2002, working on experimentally characterizing waveform, duty cycle, and frequency dependency aspects of dither.

Journal Publications

Jeff Badertscher, Kenneth A. Cunefare, "Experimental Investigation of Dither Control on Effective Braking Torque", CD-Proceedings, 2003 SAE NVH Conference. Also, invited for presentation at SAE World Congress, Detroit, MI, 2004, p. 1617, vol. 1, (2003). Published

Ferri, A.A., and Whiteman, W.E., "Stability Analysis of a Vibrational System Subject to Negative Viscous Damping and Displacement-Dependent Dry Friction Damping", Proceedings of the 2003 ASME Design Engineering Technical Conference, p. , vol. , (2003). Submitted

Badertscher, J, Cunefare, K. A., and Flynn, M., "Braking impact of normal dither signals for squeal suppression", The International Symposium on Active Control of Sound and Vibration, Active 04, Williamsburg, VA, 2004. CD Proceedings, paper A04-036., p. 1-12, vol. , (2004). Published

Books or Other One-time Publications

Web/Internet Site

URL(s):

<http://www.me.gatech.edu/acoustics/IAL/projects/michaux/index.html>

Description:

Theoretical developments, maintained by participant Michael Michaux

Other Specific Products

Contributions

Contributions within Discipline:

This project has developed improved understanding of the impact of dither control upon brake systems. Dither, though, is a generic technique for stiction suppression, such that the findings here may be relevant elsewhere.

Further, we have used dither as a 'probe' to determine if the various proposed squeal-mechanisms are amenable to dither suppression. For example, we have determined that the 'follower-force' model for squeal is not controllable by dither. However, since our experimental program does demonstrate the efficacy of dither, it calls into question the validity of the follower-force model.

Contributions to Other Disciplines:

Dither control is applied beyond mechanical system control, such as electronics. Our developments, particularly as it relates to burst mode dither, may therefore have broader relevance.

Contributions to Human Resource Development:

During this reporting period, four graduate students and one undergraduate student were involved in aspects of the project, ranging from simple and complex experimental programs, design analysis of actuator alternatives, to analytical developments and numerical simulations.

Prior reporting period (2002-2003), training continued with two graduate students in engineering research at various levels. Training of one graduate student in teaching and class supervision skills. Exposure of 2 undergraduates, through special topics classes focused on the project, to engineering research. Exposure of 2 visiting scholar undergraduates to experimental research. Exposure of approximately 26 undergraduate students, through senior capstone lab class, to concepts of dither control and squeal.

Prior reporting period (2001-2002): Training of three graduate students in engineering research at various levels. Training of one graduate student in teaching and class supervision skills. Exposure of 2 undergraduates, through special topics classes focused on the project, to engineering research. Exposure of 20 undergraduate students, through lab class, to concepts of dither control and squeal.

Contributions to Resources for Research and Education:

As noted previously, the experiment rig has been used extensively for class instruction. The experiment is providing hands-on experience and transfer of cutting edge research developments to senior undergraduates.

An NSF 'Research Experience for Undergraduates' was been approved for this project. REU funds were used to further enhance undergraduate participation in the project, starting fall semester 2003.

Contributions Beyond Science and Engineering:

Efforts have been made to interest automotive companies in the technology. A collaboration with MTS and in-kind donation from TRW has resulted from these efforts. The concepts and results have been presented to Akebono, Delphi, Ford, and General Motors. Also, we have been contacted by Federal Mogul. Additional contacts were made by potential end-users of the technology (MARTA, NY Transit Authority) and potential extensions of the technology into the railroad industry.

Special Requirements

Special reporting requirements: None

Change in Objectives or Scope: None

Unobligated funds: less than 20 percent of current funds

Animal, Human Subjects, Biohazards: None

Categories for which nothing is reported:

Organizational Partners

Any Book

Any Product

Analytical Effort

As discussed in last year's annual report, the analytical effort has simultaneously pursued the study of two different types of brake-rotor models. The first model combines a multi-degree-of-freedom (MDOF) brake rotor with a single-degree-of-freedom (SDOF) pad/caliper model. The rotor is modeled as an annular plate whose vibratory response is approximately described by a number of clamped-free annular plate modes. This model is designed to examine the instabilities that arise when system vibrational modes coalesce, or when dither frequencies parametrically excite a system mode. The second type of model developed is a single-degree-of-freedom spring-mass-damper system sliding on a moving belt. This model, although of low dimension, is ideal for the development of analytical and numerical techniques and for the examination of different types of friction laws. The results pertaining to each type of model are described below.

1. SDOF model

The single-degree-of-freedom model consists of a stationary lumped mass restrained by a linear spring and viscous damping element. The mass is pressed into contact with a rigid moving surface. A simulation program has been developed that (a) accommodates a wide variety of friction laws (b) accounts for the influence of dither forces imparted directly to the mass in the normal and/or tangential direction, and (c) can model the case of a high-frequency speed variation in the supporting belt. In order to model sticking episodes, the program switches between slipping and sticking models based on kinematic and dynamic-equilibrium criteria.

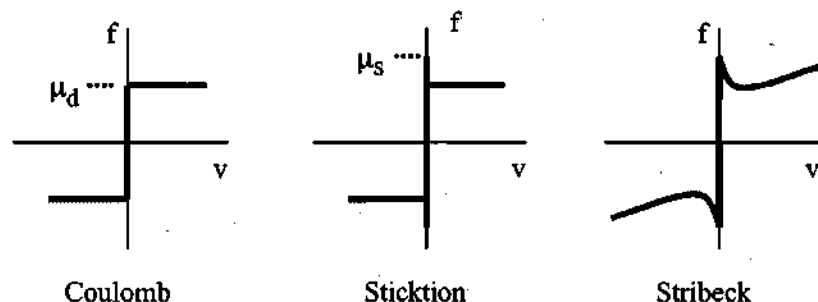


Figure 1. Coulomb, Sticktion, and Stribeck friction laws.

For the initial phase of the study, three friction laws were considered as displayed in Figure 1. The Coulomb law is characterized by having a friction force, f , that depends on a single friction coefficient which is independent of the relative slip velocity, v . The "sticktion" model accounts for the fact that most frictional interfaces have differing static (μ_s) and dynamic (μ_d) coefficients of friction. Finally, the Stribeck model, which is phenomenologically based on the tribology of *lubricated* contact, displays a friction coefficient that continuously varies with relative slip velocity. Due to its highly discontinuous nature, the sticktion model is extremely challenging to study analytically. The Stribeck model is somewhat more amenable to analysis, but may not be as appropriate for the *unlubricated* brake-pad/rotor interface. Although, in some sense, a limiting case of the Stribeck

model is the sticktion model, it is found that the performance of various dither control strategies is notably different.

Accomplishments:

- Numerical studies of tangential dither have shown the combinations of dither frequency and amplitude for which self-excited stick-slip oscillations are suppressed. It was found that the dither amplitude required to suppress the sustained oscillations decreases with dither frequency; i.e., high frequency dither signals (relative to the system resonant frequency) seem to be the most effective.
- It was found that, for a given dither frequency, the dither amplitude required to suppress sustained oscillations follows a linear relationship with the ratio of static coefficient of friction over dynamic coefficient of friction, μ_s/μ_d . In other words, as μ_s/μ_d approaches 1, the required amplitude of the tangential dither signal decreases.
- Analytical studies of the SDOF model have been performed based on the techniques developed by Jon Thomsen and co-workers at the Technical University of Denmark. The technique uses an averaging technique to approximate the response to first order as a superposition of "fast" and "slow" response. The extension of the technique to sticktion models with tangential dither and to the general case of normal dither is being studied.
- The ability of friction to suppress instabilities in SDOF systems with negative viscous damping was studied. Although not directly applicable to the problem of brake squeal cancellation, the developed methodology of utilizing piecewise linear dynamics to obtain exact solutions for limit cycles was used as the basis for obtaining exact solutions for the case of tangential dither excitation of the sticktion model.
- In addition to sinusoidal dither signals, initial numerical investigations using different types of dither signals have been conducted. The simulation program has been given the capability to handle harmonic, triangular, and square waveforms as well as off-on "burst modes" with adjustable duty cycles.

2. Brake-rotor/brake-caliper model

While the SDOF model has allowed a close inspection of the interplay between the dither forces and the friction models, it cannot be used to determine the influence of multiple system modes. In particular, it is known that even in the case of steady-sliding in the absence of friction, instabilities can arise in rotating systems. This year, the model of a rotating annular plate and a stationary spring-mass-damper (SMD) system was further developed. The plate interacts with the SMD system in two ways: a transverse force that is independent of friction, and a constant follower force that is due to friction. In the case of no-dither, the follower force is of constant magnitude, but its direction is always tangent to the local deformation of the plate. Thus, the follower force is nonconservative and can contribute to system instabilities. It is important to stress that the instabilities occur without the need of stick-slip oscillations. Therefore, the resulting equations of motion are *linear*.

Accomplishments:

- A sixth-order model was developed involving the coupled dynamics of three plate modes. In the case of harmonic dither, the time-periodic system resembles three coupled Mathieu equations. The stability of this system was studied using Floquet theory.
- It was found that small changes in system stability could be realized when the system was in a parameter range near a stability boundary. This investigation is ongoing and has yet to be compared qualitatively with the observed squeal suppression characteristics exhibited by the experimental brake dynamometer system.

Activities – Experimental Effort

The general objectives of the experimental program, addressed during the reporting period, were to determine the impact of dither control on mean braking torque. Experiments are being designed to show the evolution of the frequency response of the braking components as the system is brought into squealing conditions.

The experimental effort has benefited from the existence of a fully functional experimental rig in service at time of funding. Since that time, the rig has been upgraded to incorporate in-line torque sensing capabilities and a new dither actuation device is being developed.

Beyond the specific experiments described below, we have implemented the means to identify squeal modes, and, have performed extensive testing to establish repeatable squeal conditions. These tasks are basic foundations to the subsequent experiments, described below (i.e., one must have a repeatable squeal in order to assess dither suppression efficacy).

1. Dither Implementation

Experiments were conducted that demonstrated dither to be an effective means of suppressing brake squeal. In these experiments, a piezo-electric stack actuator located in the brake piston produced the dither signal. To improve on the range of dither forces and clarity of the dither signal a new housing for the stack is being developed. The new system will provide the stack with a preload to ensure the stack is always in compression and improve force transmission to the brake pads.

2. Torque Experiments

The results of torque testing to date indicate that a continuous dither signal of 23 kHz and varying force amplitudes reduces the effective braking torque by 3 to 5%. These results suggest that perhaps the Hess and co-workers model is a good starting point but that it overestimates the impact that dither control has on braking torque.

The fact that the brake pressure has no effect on the reduction in braking torque is a very substantial result. The only effect brake pressure has on the system is the ability of the piezoceramic stack to

be driven at high force amplitudes. Increasing the brake pressure does not increase the dither penalty on the braking torque. Therefore, at constant dither forcing, emergency braking is no more impacted than light braking.

The impact of dither control on the effective braking torque is directly related to the force amplitude of the dither signal. This result could prove to be the most significant if a threshold value of dither force amplitude is established for torque reduction.

Current work in progress involves establishing the force threshold for torque reduction and relating this threshold to the force threshold for suppression of brake squeal.

3. Modal Identification

Previous work was done to determine the modal behavior of the brake assembly. Future work is planned to determine how the frequency response changes as the system is brought into and out of squealing conditions. This work will help to validate or refute the analytical models that attribute brake squeal to a mode coalescence phenomenon.

4. Education Activities

To date, approximately 50 undergraduate students have used the brake squeal dynamometer as part of their senior undergraduate laboratory class. The objectives of their lab experience has paralleled the research objectives of the project. For example, in the spring of 2002, groups investigated the modal behavior of the rotor/caliper system while static and while bringing the system into squeal. In the summer of 2002, lab groups investigated the impact of dither and various dither waveforms on mean brake torque. Projects in fall 2002 and spring 2003 focused on the impact of dither on mean torque.

One graduate student participating in the project (supported through other means) has gained teaching training through supervision and delivery of the laboratory class.

Four undergraduate students have participated in the research, through special topics classes supervised by Dr. Cunefare. They have been performing tests to maintain a steady brake squeal and investigating the effect of various waveforms on the mean braking torque.

Two foreign exchange students worked on the brake squeal project for a semester. Their research was to examine the nature of control using a burst modulated dither signal, specifically looking at the squeal signal during the dwell time of the burst.

5. Major Publications

Jeff Badertscher, Kenneth A. Cunefare, "Experimental Investigation of Dither Control on Effective Braking Torque," CD-Proceedings, 2003 SAE NVH Conference (refereed proceedings), paper 2003-01-1617, May 2003.

Continuous Model

An annular plate model, described in previous reports, was further studied this year. The model includes several plate modes, and includes a rotating spring-mass-damper (SMD) system that maintains contact with the rotor's surface at all times. Assuming steady sliding, the brake friction enters into the model as a follower force, i.e., a force that is always tangential to the rotor surface. When the friction normal force is of constant magnitude, the governing equations are linear and time-invariant (LTI.) When dither is introduced, the equations become linear and time-varying (LTV) with periodic coefficients. The onset of brake squeal is determined by the eigenvalues of the LTI system, or using Floquet theory applied to the LTV system.

Stability studies conducted this past year revealed that the frictional follower force is highly destabilizing. By increasing the levels of viscous/structural damping in the system, subcritical regions of stability could be created. However, the level of damping required was not physically justified. Another deficiency of the model is that it predicts critical speeds that are much higher than the rotor speeds that one typically encounters during braking. Perhaps this type of instability mechanism is more prevalent in computer-disk systems where rotation speeds are higher, and where the moving loads are more concentrated. Brake pads tend to distribute the loads over regions of the brake rotor; previous researchers have found that this may lessen the instability problems.

Recent work by Mottershead and co-workers has found that a rotating couple (as opposed to a rotating transverse force) can lead to low-speed instabilities. This leads us to consider more realistic models of the caliper/brake-pad system. This work is further described below.

SDOF Lumped Model

Single-degree-of-freedom (SDOF) models have often been used to describe stationary frictional systems in contact with moving surfaces. As described in the last annual report, a SDOF model was developed that accommodates many different friction models, and incorporates tangential and normal dither. Numerical simulations of this system continued this year, and a journal paper documenting these findings is in preparation.

An averaging technique described by Thomsen, was applied to the SDOF system with tangential dither. The analytical technique is restricted to the case of stick-free motion, avoiding the discontinuities in the friction law at the point of zero slip velocity. The accuracy of the technique is being studied through comparisons with numerical simulations. In particular, the influence of the "shape" of the friction law on the accuracy of the technique is being assessed.

MDOF Modeling

While SDOF models are helpful from a qualitative standpoint, more refined analyses require the use of multi-degree-of-freedom (MDOF) models. The initial model chosen for study was developed by Earles (1987). The model includes contact on both the inboard and outboard side of the brake rotor. Each contacting body is modeled using three degrees of freedom: two translations and one rotation. The rotor is modeled as a lumped mass with transverse displacement only. Two constraints are

used to ensure that continuous contact is maintained at all times. Numerical studies have been completed in order to validate the model against published results.

Missing from the Earles model is the correct kinematics for the rotor vibration. This deficiency will be corrected by using two to three modes of the annular plate to capture realistic coupling between rotor transverse and rotational movement. Also needed is a caliper model that connects the front and back lumped-mass assemblies together.

Numerical/Experimental Studies

With the rotor firmly mounted on the hub, the modes of the rotor in the experimental setup were measured and classified according to the number of nodal diameters observed. Comparisons between the squeal vibration signal and that of the SDOF model revealed an interesting correlation. The Fourier spectrum of the slip velocity of the SDOF system shows that the response is dominated by a fundamental frequency, which is close to the natural frequency of the SDOF system, and the second harmonic. Also present, are successively smaller amounts of the third, fifth, seventh, etc., harmonics of the fundamental frequency. This same frequency distribution was observed in the laser probe of the rotor vibration during squeal. Specifically, the squeal vibration signature was found to be dominated by frequencies of 2.82, 5.703, and 8.547 kHz. It is seen that the second frequency is approximately twice the fundamental, while the third is approximately 3 times the fundamental. Another important observation is that the measured frequency components match the natural modes of the mounted rotor.

This finding suggests that the brake squeal in this particular experimental setup is consistent with stick-slip oscillations of the brake pads against the rotor, giving rise to a periodic vibration. The dominant frequency components in the resulting periodic limit cycle vibration coincide with brake rotor natural frequencies, thereby reinforcing the response. Further measurements and analysis are necessary to confirm whether stick-slip occurs between the brake pads and the rotor, or whether partial sticking occurs over some portion of the contact area.

SDOF models do not easily provide a mechanism by which tangential motion of the caliper/brake-pad assembly can be coupled to the transverse vibration of the rotor. It is possible that "rocking" motion of the caliper assembly is the means by which tangential motion couples into the transverse motion of the rotor. Based on this assumption, MDOF models of the rotor-caliper system are being developed for numerical study.

In an effort to simplify the experimental setup, and to isolate the influence of the friction law on friction-induced oscillations, a SDOF experimental setup is being developed. Initially, the setup will implement tangential dither, and the results will be compared against those of the analytical averaging technique as well as those from numerical simulations.

Extensive experimental work focused on assessing the braking torque impact of dither. The question of interest was to what extent the brake torque may increase, or decrease, with dither applied. A number of confounding physical responses must be accounted for in this assessment. For example, throughout a braking event, the temperature of the rotor and pads continuously increases. As the coefficient of friction between the pads and rotor may vary with temperature, then time-dependent

temperature effects must be extracted from any torque variation that occurs over time. In addition, the brake pressure itself may vary over time, due to the nature of the control system acting on the brake lines. Torque variations due to this effect must also be subtracted. To account for these effects, the test protocol was structured such that temperature effects could be isolated and extracted, as well as line pressure variations. With these effects subtracted, it was found that the torque with dither applied was only slightly different from that without dither, such that the sample-to-sample variation itself become the next factor of interest, requiring a high number of repetitions (typically 50) in order to achieve statistically valid results.

Tests were performed to assess torque impact as a function of brake line pressure, dither force amplitude, and dither frequency. Tables 1, 2 and 3 present results for these programs, indicating that dither does generally yield a reduction in brake torque, with the largest reduction being on the order of 3% (as compared to the same braking conditions without dither); this magnitude of reduction may be judged to be relatively insignificant, and may be readily compensated for in automotive practice by proportionally greater brake pedal pressure.

Table 1. Torque Impact for Selected Line Pressures and Rotor Speeds

Brake Pressure (MPa)	Motor Speed (rpm)	Vehicle Speed (mph)	Torque Impact
0.2068	1100	3.5	-0.88 to -0.65 %
0.6205	1100	3.5	-0.89 to -0.61 %
1.0342	1100	3.5	-0.55 to 0.38 %
0.6205	1000	3.2	-1.07 to -2.70 %
0.6205	1100	3.5	-0.89 to -0.61 %
0.6205	1200	3.8	-0.13 to -1.02 %

Table 2 Torque Impact as Function of Dither Force Amplitude

PZT Voltage	Dither Force (N)	Torque Impact
20	50	0.48 to -0.29 %
65	100	-0.32 to -0.49 %
100	150	-0.24 to -1.33 %
125	200	-0.89 to -0.61 %
150	250	-1.46 to -1.38 %

Table 3 Torque Impact as Function of Dither Control Frequency

Frequency (khz)	Dither Force (N)	Torque Impact
4.6625	200	0.46 to -0.24 %
6.3000	200	0.17 to -0.81 %
12.0000	200	-0.30 to -1.19 %
19.2500	200	-0.612 to -1.98 %
25.6000	200	-0.89 to -0.61 %